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Annual Report

GIMBALLESS INERTIAL NAVIGATION SYSTEMS

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ANNUAL REPORT

STUDY OF GIMBALLESS INERTIAL NAVIGATION SYSTEMS

1. INTRODUCTION

This report on a study of gimballess inertial navigation systems summarizes the work performed during the first year of this study. The report is quite brief since the details of most of the work are given in the following publications.

"Damping Gimballess Inertial Navigation Systems," by A. Grammatikos, A. R. Schuler, and K. A. Fegley, Conference Record of the Twelfth I.E.E. East Coast Conference on Aerospace and Navigational Electronics, October, 1965. (This paper was selected by the Technical Program Committee for first prize award.)

"Measuring Rotational Motion With Linear Accelerometers," by A. R. Schuler, A. Grammatikos, and K. A. Fegley, Conference Record of the Twelfth I.E.E.E. East Coast Conference on Aerospace and Navigational Electronics, October, 1965.

Gimballess Inertial Systems for Space Navigation, Ph.D. dissertation in electrical engineering by Antonios Grammatikos, University of Pennsylvania, Philadelphia, Pa., 1965.

Design and Analysis of Analytic Platform Inertial Navigation Systems, Ph.D. dissertation in electrical engineering by A. R. Schuler, University of Pennsylvania, Philadelphia, Pa., 1965.

The work accomplished can be conveniently divided into the six areas: (1) all accelerometer techniques for measuring linear and rotational parameters of motion, (2) damping of the navigation loop, (3) the transformation matrix, (4) error analyses, (5) computer simulation, and

(6) experimental work. A summary of the work in each of these six areas is given in the following section.

2. SUMMARY OF WORK

2.1. All accelerometer techniques for measuring linear and rotation parameters of motion

At present, inertial navigation systems usually use gyroscopes to sense angular motion and accelerometers to sense linear accelerations. It is possible, however, to determine both linear acceleration and angular motion by using six or more accelerometers that are fixed to the vehicle or by using three accelerometers with one mounted on each of three mutually perpendicular rotating rings.

A number of fixed accelerometer configurations have been investigated. The most useful of these configurations are discussed in References 2, 4, 5, and 6. Reference 4, while not the most complete, gives the most concise discussion. Reference 5 gives the most complete discussion.

While it might be expected that six fixed accelerometers would provide sufficient information to find the three components of linear acceleration and the three components of angular motion for a six-degree-of-freedom vehicle, this has not been found to be true. One particular six-accelerometer configuration provided the three components of linear acceleration and the three components of angular velocity, but with a sign ambiguity for the angular velocity components. For another six-accelerometer configuration, the equations for the angular motion are unstable. No completely satisfactory configuration of six fixed accelerometers has been found.

A number of configurations having six, seven, eight, or nine fixed accelerometers have been studied and the relative merits of the several configurations have been discussed. For all of the configurations considered, the accelerometers lie along mutually perpendicular axes which coincide with or are parallel to the principal axes of the vehicle, and the sensitive axis of each accelerometer is along or normal to the axis along which it lies.

Due to fuel consumption, the center of mass of the vehicle may move within the vehicle. This motion of the center of mass due to fuel consumption need cause no loss in the accuracy with which the angular and linear motion of the vehicle are determined if 1) the motion of the center of mass within the vehicle is accurately known and 2) the principal axes of the vehicle remain parallel to the axes along which the accelerometers are mounted.

Krishnan[3] proposed using rotating accelerometers to determine both linear and angular motion of the vehicle. He suggested using three concentric and mutually perpendicular rings, each carrying an accelerometer, and mounted with their centers at the center of mass of the vehicle. It has been possible to reduce the mounting restrictions[5], i.e., 1) it is not necessary that the rotating rings be at the vehicular center of mass, and 2) it is not necessary that the rings be concentric. If the vehicle always has spin about at least one axis, then two rotating accelerometers rather than three will suffice.

2.2. Damping the navigation loop

In the navigation loop, the three components of acceleration are each integrated twice to find the three components of position. This calculated position is used to compute the gravity acceleration which is to be subtracted from the measured acceleration. This double integration together with the feedback provided by the gravity computation yields an unstable system. Measurement errors, however small, are unavoidable. These errors enter the navigation loop and appear as oscillatory or diverging errors in velocity and position.

The velocity and position errors in the navigation loop can be damped by introducing into the navigation loop velocity or positional information from auxiliary sensors such as optical- or radar-Doppler or stellar trackers. For vehicles that approximately follow a reference trajectory, the positional information of this reference trajectory can be used to damp the errors of the navigation loop.

Methods of damping the navigation loop are presented in References 1, 2, 5, and 6. Reference 1 gives a concise discussion of damping techniques which are also presented in Reference 2; Reference 5 presents a different scheme which has many similarities with the others.

In all of the navigation loop damping techniques which have been investigated to date, the equations have been linearized by Taylor series expansion and it has been assumed that components of position are relatively constant with respect to position and acceleration errors.

2.3. The transformation matrix

In an all-accelerometer gimballess inertial navigation system, the translational and rotational motions detected by the accelerometers are inertial quantities referred to vehicular coordinates. It is generally advantageous to refer these inertial quantities to an inertial frame before inserting gravity correction and before integrating to obtain vehicular velocity and position. To refer the measured quantities to the inertial frame, a direction cosine matrix is used.

The equations of the direction cosine matrix can be written as nine differential equations, with three of the nine equations being independent. Analysis of these equations in the neighborhood of a nominal operating point shows that they are oscillatory. Solution of the direction cosine matrix by using difference equations leads to diverging errors with the direction cosine matrix maintaining neither normality nor orthogonality.

Auxiliary position information from star trackers may be introduced to "damp" the oscillations in the direction cosine loop.[1,2] To maintain the normality and orthogonality of the direction cosine matrix, an algorithm that uses all nine of the direction cosine equations has been developed.[5] It has also been shown[5] that an exact closed form solution for the direction cosines can be found if the rate of change of angular acceleration of the vehicle approaches zero.

2.4. Error analyses

The error analyses have had several facets as indicated by the following paragraphs.

The measurements obtained from the accelerometers contain errors. By suitable filtering of these errors, the accuracy of the navigation system can be improved. An optimum digital filter (in the sense of minimizing output noise variance) has been developed[5] for correlated and for uncorrelated input noise. A computer study has verified the performance of this filter.

The characteristics of the errors in the accelerometer measurements can be expressed in terms of their autocorrelation functions. Based upon the autocorrelation functions of the accelerometer outputs, the autocorrelation functions have been found for the errors in linear acceleration and angular velocity.[5] These autocorrelation functions are for linear accelerations referred to both vehicular and inertial frames and for angular velocities referred to the vehicular frame. For cases in which the resulting autocorrelation functions of the errors are non-stationary, the variance of the error output as a function of time is determined.

Equations are derived for the errors that arise when the accelerometers are misaligned and the problems associated with computational and initial alignment errors are discussed.[5]

In addition, the errors in the computation of gravity in the navigation loop and the errors in the computation of the direction cosine matrix are considered.[2,5]

2.5. Simulation studies

A navigation system has been simulated on a digital computer. This simulated system includes 1) a configuration of six fixed accelerometers displaced from the center of mass, 2) the direction cosine transformation matrix to refer accelerations to the inertial axes, 3) a linear filter to estimate the "best" values of acceleration, and 4) the navigation loop, including a damping circuit and a gravity computer.

The input to the system is an angular velocity pattern and a linear acceleration pattern. The input is used to determine the "exact" or reference trajectory of the vehicle. The components of the input are also combined to give the six accelerations which the six fixed accelerometers read. Noise is added to these six accelerations to simulate the actual accelerometer measurements. The outputs from the noisy accelerometers are filtered and then algebraically combined to obtain the best estimate of angular velocity and linear acceleration. The components of the linear acceleration are then referred to the inertial axes by the direction cosine matrix. The computed gravity, based upon the calculated position of the vehicle, is subtracted from the computed acceleration. A double integration of this modified acceleration is then performed in a navigation loop which includes damping. The auxiliary information required for damping the navigation loop is supplied by a star tracker. The output from the star tracker is also simulated on the computer.

To date, the undamped system has been studied for an error free system and for a system with gaussian accelerometer errors having no correlation between sample values. The linear filter has been used for some of the runs where the accelerometers were noisy. The filtering has not been completely satisfactory. It is anticipated that improved results can be obtained by filtering after the accelerometer outputs have been algebraically combined.

Simulation runs have also been made for the direction cosine transformation matrix alone and drift calculations due to computer roundoff and angular velocity errors have been made.

In summary, the simulation has been programmed but has not been thoroughly checked or run on closed loop with all subroutines in operation. It is anticipated, however, that this simulation will be of inestimable aid in designing gimballess inertial navigation systems.

2.6. Experimental work

An experimental study to demonstrate the feasibility of and associated problems of a rotating accelerometer sensing system is being pursued. A rotating disk, mounted on a dividing head and driven by a Ward-Leonard speed control system has been constructed. A loop-closing amplifier has been constructed and the accelerometer and amplifier have been operated. The two major problems that have become apparent are: 1) the limitation in the transverse acceleration which the accelerometers can withstand and 2) the noise generated by the sliding contacts.

The Bell III-B accelerometers which are being used for the experimental work can withstand a 20 g (perhaps even 100 g) acceleration that is transverse to its sensitive axis. One accelerometer, however, was damaged when subjected to a tangential acceleration of approximately 200 g.

The original slip rings were flat concentric rings mounted on one face of the rotating disk. Graphite brushes were used. These sliding contacts have been unsatisfactory and are to be replaced by precious metal rings and brushes. To keep the velocity of the sliding contacts low, rings of small diameter will be used.

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